A Report on "Targeting Interventions in Networks" by Galeotti et al. (2020)

Reviewer 2

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> I am wiser than this person; for it is likely that neither of us knows anything fine and good, but he thinks he knows something when he does not know it, whereas I, just as I do not know, do not think I know, either. I seem, then, to be wiser than him in this small way, at least: that what I do not know, I do not think I know, either.
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> Plato, *The Apology of Socrates*, 21d

> To err is human. All human knowledge is fallible and therefore uncertain. It follows that we must distinguish sharply between truth and certainty. That to err is human means not only that we must constantly struggle against error, but also that, even when we have taken the greatest care, we cannot be completely certain that we have not made a mistake.
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> Karl Popper, ‘Knowledge and the Shaping of Reality’

Overview

Citation: Galeotti, A., Golub, B., and Goyal, S. (2020). Targeting Interventions in Networks. *Econometrica*, Vol. 88, No. 6, pp. 2445–2471.

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Abstract Summary: This paper examines how a planner can optimally target interventions in network games to influence individuals' private returns to investment. It decomposes interventions into orthogonal principal components determined by the network and ordered by associated eigenvalues, revealing a connection between spillover nature and optimal intervention structure.

Key Methodology: The authors use a principal component decomposition approach to analyze the effects of interventions on social welfare in a simultaneous-move game with continuous actions, subject to a budget constraint on intervention costs.

Research Question: How does a planner optimally target interventions that change individuals' private returns to investment in games where a network mediates strategic spillovers and externalities among players?

Editor's Note

The reviewer provides a rigorous critique, particularly regarding the mathematical proofs in the supplementary material and the practical limitations of the linear-quadratic framework. To defend the work, it could be helpful to remind the reviewer that the article's primary goal is to establish a foundational theoretical benchmark, not a ready-to-deploy policy manual. The linear-quadratic framework, symmetric costs, and the assumption of a unique equilibrium are standard in this literature and are necessary to cleanly isolate first-order spectral effects. The large-budget asymptotic results could be defended as mathematical limits that reveal the underlying directional pull of the network's spectral properties, rather than literal prescriptions that ignore non-negativity constraints. The reviewer's conceptual concerns might be satisfied by adding a dedicated discussion section that explicitly acknowledges these practical limitations, such as the informational burden of computing global eigenvectors and the assumption of symmetric costs, framing them as natural next steps for future research.

A pressing task is to correct the mathematical and algebraic errors identified in the supplementary material. The divergent limit in the proof of Proposition OA1 could be resolved by correcting the scaling factor to ensure Berge's theorem of the maximum applies. Similarly, the logical contradiction in Theorem OA1, where a limit to infinity is taken for a budget that is strictly bounded from above, could be fixed by explicitly restricting the asymptotic results to the case where the relevant coefficient is non-negative. Furthermore, the numerous algebraic slips, such as the incorrect derivation of aggregate welfare coefficients, the erroneous square in the singular value decomposition equilibrium action, and the missing parameters in the utility definitions, would benefit from correction. Fixing these does not undermine the core argument but restores the technical credibility of the robustness checks.

If correcting the divergent limit in Proposition OA1 or the asymptotic limits in Theorem OA1 proves too mathematically complex without fundamentally altering the proofs, an alternative plan could be considered. In this fallback approach, the ambition of the supplementary material could be scaled back. The generalized cost function analysis could be restricted solely to small budgets, explicitly abandoning the large-budget claims for the generalized cases where the limits break down. Additionally, the problematic asymptotic claims in Theorem OA1 could be removed entirely, focusing only on the exact characterizations. This compromise sacrifices some theoretical breadth but ensures the published proofs are mathematically airtight and avoids a protracted battle over the supplementary extensions.

Summary

Is It Credible?

Galeotti, Golub, and Goyal provide a theoretical framework for targeting interventions in network games. The authors claim that the effects of an intervention can be elegantly decomposed using the principal components, or eigenvectors, of the network's interaction matrix. Their headline claims establish a direct link between the nature of strategic interactions and the optimal targeting strategy. Specifically, they argue that in games with strategic complements, optimal interventions prioritize the top principal components, which reflect the global network structure. Conversely, in games with strategic substitutes, interventions prioritize the bottom principal components, which reflect local network structure. Furthermore, the authors claim that for sufficiently large budgets, optimal interventions become "simple," meaning they are proportional to a single principal component.

The core insight linking strategic interactions to spectral properties is mathematically elegant and provides a novel conceptual framework for understanding network spillovers. However, the credibility of the headline claims as practical policy guides is limited by several strong assumptions and mathematical fragilities. The entire approach relies heavily on a linear-quadratic framework, which guarantees linear best responses and a unique, stable equilibrium. This precludes the study of tipping points or multiple equilibria, which are often central to the study of network interventions. Furthermore, the model assumes a symmetric quadratic cost function where taxing and subsidizing have identical cost structures, abstracting away from the vastly different administrative and political realities of these tools.

The claim that optimal interventions become simple for large budgets faces significant economic and mathematical hurdles. For strategic substitutes, targeting the lowest eigenvector involves assigning opposite incentives to adjacent nodes. For large budgets, this would force massive negative shifts in standalone returns. While the authors briefly note that non-negativity constraints on actions are respected for small budgets (p. 2455), a large-budget intervention would inevitably drive equilibrium actions into negative territory, violating standard economic assumptions. Furthermore, the threshold for what constitutes a large budget can be astronomically high, scaling inversely with the squared spectral gap of the network. As the authors acknowledge in a footnote, this threshold can be orders of magnitude larger than the status quo returns (p. 2456). Consequently, for any realistic policy budget, the optimal intervention will likely remain a complex mixture of components rather than a simple, single-eigenvector policy.

The article attempts to generalize its findings to broader cost structures and non-symmetric networks in the supplementary material, but these extensions contain mathematical and logical errors that undermine their reliability. For example, the proof extending the results to general cost functions relies on a divergent limit that invalidates the subsequent application of the theorem of the maximum. Additionally, the asymptotic results in the supplementary theorems contain a logical contradiction by taking a limit as the budget approaches infinity while simultaneously assuming the budget is strictly bounded from above. Numerous algebraic errors in deriving welfare coefficients and cost formulas further weaken the credibility of these robustness checks.

Finally, the informational requirements for these simple interventions are paradoxically complex. Computing global eigenvectors requires perfect, centralized knowledge of the entire network topology. The analysis does not account for measurement error or network noise, to which spectral methods are highly sensitive. Moreover, the article does not benchmark its optimal spectral policy against simpler, localized heuristics, making it difficult to assess the practical value of its complex informational demands. Ultimately, while the article offers a genuinely novel mathematical decomposition of network interventions, its findings should be viewed as abstract theoretical benchmarks rather than deployable policy prescriptions.

The Bottom Line

Galeotti, Golub, and Goyal provide an elegant theoretical framework that maps optimal network interventions to the principal components of the interaction matrix. While the core insight linking strategic complements and substitutes to global and local network structures is conceptually sound within the model's linear-quadratic constraints, the practical implications are limited. The claim that optimal interventions become simple for large budgets is undermined by unrealistic budget thresholds, the violation of non-negativity constraints on actions, and mathematical errors in the supplementary robustness checks. Ultimately, the article represents a significant methodological contribution to network theory, but its heavy reliance on perfect information and restrictive functional forms limits its immediate applicability to real-world policy design.

Potential Issues

Divergent limit in the proof of Proposition OA1: In the proof of Proposition OA1 (p. S14), the re-scaled objective function is defined as $C^{-1}\Delta(C^{1/2}\breve{y})$. The change in welfare is $\Delta(y) = w\sum \alpha_\ell(2\underline{\hat{b}}_\ell \underline{y}_\ell + \underline{y}_\ell^2)$. Substituting $y = C^{1/2}\breve{y}$ yields $\Delta(C^{1/2}\breve{y}) = w\sum \alpha_\ell(2\underline{\hat{b}}_\ell C^{1/2}\breve{y}_\ell + C\breve{y}_\ell^2)$. Multiplying by C^{-1} gives $2wC^{-1/2}\sum \alpha_\ell \underline{\hat{b}}_\ell \breve{y}_\ell + w\sum \alpha_\ell \breve{y}_\ell^2$. As $C \to 0$, the term $C^{-1/2}$ diverges to infinity. Consequently, the limit function $F(\breve{y})$ is mathematically undefined, which invalidates the subsequent application of Berge's theorem of the maximum. To achieve a well-defined limit, the objective should have been scaled by $C^{-1/2}$.

Incorrect derivation of aggregate welfare coefficients: In Section OA3.1 (p. S6), the coefficients w_1 and w_2 for the aggregate equilibrium utility $W(b, G)$ are derived incorrectly. The utility is $U_i = \hat{U}_i + P_i$, where $\hat{U}_i = a_i(b_i + \beta\sum_j g_{ij}a_j) - \frac{1}{2}a_i^2$. At equilibrium, $\hat{U}_i = \frac{1}{2}a_i^2$. Summing over i gives $\frac{1}{2}\sum_i a_i^2$. For the externality term P_i, the term $m_4\sum_i(\sum_{j\neq i} a_j)^2$ evaluates to $m_4((n-2)S^2 + \sum_i a_i^2)$, and $m_5\sum_i\sum_{j\neq i} a_j^2$ evaluates to $m_5(n-1)\sum_i a_i^2$. The total coefficient of $\sum_i a_i^2$ is therefore $\frac{1}{2} + m_2 + m_4 + m_5(n-1)$, and the coefficient of S^2 is $m_4(n-2)$. The text incorrectly states $w_1 = 1 + m_2 + m_5 + (n-1)m_4$ and $w_2 = nm_5(n-2)$. The base coefficient for $\sum_i a_i^2$ should be 1/2, not 1, and the coefficient w_2 should utilize the parameter m_4, not m_5.

Invalid limit in Theorem OA1: Parts 4 and 5 of Theorem OA1 (p. S8) describe the asymptotic behavior of the optimal intervention as the budget $C \to \infty$, stating that $\mu \to \max\{w_1\alpha_2, (w_1 + w_2)\alpha_1\}$. However, the premise of the theorem states: "Suppose that either (i) $w_1 \geq 0$ or that (ii) $w_1 < 0$ and $\sum_{\ell=2}\underline{\hat{b}}_\ell^2 > C$." If $w_1 < 0$, the budget C is strictly bounded from above by $\sum_{\ell=2}\underline{\hat{b}}_\ell^2$, which means the limit $C \to \infty$ cannot be taken. The theorem contains a logical contradiction by failing to explicitly restrict the asymptotic results in parts 4 and 5 to the case where $w_1 \geq 0$.

Algebraic error in the cost of intervention formula: In Section OA3.4 (p. S19), the formula for the cost of intervention $K(y)$ contains an algebraic error. The formula is given as $K(y) = \frac{1}{2}\sum_i \mathbf{1}_{y_i>0}\int_{a_i(y)-y_i}^{a_i(y)} s_i^1(\tau_i)d\tau_i + \sum_i(1-\mathbf{1}_{y_i>0})\int_{a_i(y)}^{a_i(y)+|y_i|} s_i^0(\tau_i)d\tau_i$. The integral for $y_i > 0$ evaluates to $\frac{1}{2}y_i^2$. Substituting this back into the first summation yields $\frac{1}{2}\sum_{y_i>0}(\frac{1}{2}y_i^2) = \frac{1}{4}\sum_{y_i>0} y_i^2$. The text claims this entire expression equals $\frac{1}{2}\sum_i y_i^2$. The leading factor of 1/2 before the first summation is erroneous and contradicts the final equality.

Arbitrary constants violating non-negativity: In Section OA3.3 (p. S14), the authors provide an example cost function $\tilde{\kappa}(y) = y^2 + c|y|^3e^y + c'y^4$ and state that c and c' are "arbitrary constants". However, Assumption OA2 part (3) requires $\kappa(y) \geq 0$ for all y. If c' is an arbitrary negative constant, as $y \to -\infty$, the e^y term vanishes and the $c'y^4$ term dominates, driving $\tilde{\kappa}(y)$ to $-\infty$. This makes the cost function negative, violating the assumption. The constant c' must be explicitly constrained (e.g., $c' \geq 0$).

Incorrect bound in feasible interval: In the proof of Theorem OA1 (p. S9), the feasible interval for x_1 is derived such that the remaining budget $C(x_1) \geq 0$. Given $C(x_1) = C - \hat{\underline{b}}_1^2 x_1^2$, the condition $C(x_1) \geq 0$ implies $x_1^2 \leq C/\hat{\underline{b}}_1^2$. Taking the square root yields $|x_1| \leq \sqrt{C}/|\hat{\underline{b}}_1|$. The text incorrectly omits the square root on C, stating the interval as $x_1 \in [-C/\hat{\underline{b}}_1, C/\hat{\underline{b}}_1]$. This error is repeated on p. S10.

Incorrect symbol in beauty contest first-order condition: In Section OA2.2 (p. S4), the utility function is $U_i(a, G) = a_i(\tilde{b}_i + \tilde{\beta}\sum_j g_{ij}a_j) - \frac{1}{2}a_i^2 - \frac{\gamma}{2}\sum_j g_{ij}[a_j - a_i]^2$. Taking the derivative with respect to a_i and simplifying yields $a_i = \frac{\tilde{b}_i}{1+\gamma} + \frac{\tilde{\beta}+\gamma}{1+\gamma}\sum_j g_{ij}a_j$. The article incorrectly writes the numerator of the second term as $\tilde{b}_i + \gamma$ instead of $\tilde{\beta} + \gamma$.

Incorrect normalization of eigenvector: In Lemma OA1 part 2 (p. S6), the first eigenvector is stated as $u_i^1(G) = \sqrt{n}$ for all i. The principal components u^ℓ are defined in Fact 1 to be orthonormal, meaning $\|u^1\| = 1$. If $u_i^1(G) = \sqrt{n}$, the norm would be $\|u^1\| = \sqrt{\sum_{i=1}^n(\sqrt{n})^2} = \sqrt{n^2} = n$, violating the normalization. The correct normalized eigenvector for a regular graph is $u_i^1(G) = 1/\sqrt{n}$.

Missing parameter in private utility definition: In Section OA3.1 (p. S5), the private utility function is defined as $\hat{U}_i(a, G) = a_i(b_i + \sum_j g_{ij} a_j) - \frac{1}{2} a_i^2$. To yield the correct equilibrium condition $a_i = b_i + \beta \sum_j g_{ij} a_j$, the equation is missing the strategic interaction parameter β in the spillover term and should be $\hat{U}_i(a, G) = a_i(b_i + \beta \sum_j g_{ij} a_j) - \frac{1}{2} a_i^2$.

Contradictory coefficient for externality term: In Example OA1 (p. S5), individual utility is $U_i(a, G) = \hat{U}_i(a, G) - \gamma \sum_{j \neq i} a_j$. Summing this over all i yields an externality term of $-\gamma(n-1) \sum_i a_i$. Therefore, aggregate welfare should be $W(b, G) = \frac{1}{2}(a^*)^\top a^* - \gamma(n-1) \sum_i a_i^*$. Equation (OA-1) incorrectly states the coefficient as $n\gamma$: $W(b, G) = \frac{1}{2}(a^*)^\top a^* - n\gamma \sum_i a_i^*$.

Incorrect definition of aggregate utility square root: On p. S17, the text states that $\|a(1_i)\|$ is "the square root of the aggregate equilibrium utility in the game with $b = 1_i$". The aggregate equilibrium utility is defined as $W(b, G) = \frac{1}{2}(a^*)^\top a^* = \frac{1}{2}\|a^*\|^2$. Therefore, the square root of the aggregate equilibrium utility is $\frac{1}{\sqrt{2}}\|a(1_i)\|$, not $\|a(1_i)\|$.

Erroneous square in SVD equilibrium action: In Section OA3.2 (p. S13), the singular value decomposition is $M = I - \beta G = USV^\top$. The equilibrium condition implies $S\underline{a}^* = \underline{b}$. Solving for the ℓ-th component yields $\underline{a}_\ell^* = \frac{1}{s_\ell}\underline{b}_\ell$. The text incorrectly writes this with an erroneous squared term: $\underline{a}_\ell^* = \frac{1}{s_\ell}\underline{b}_\ell^2$.

Missing factor in limit ratio: In Theorem OA1, part 3 (p. S8), the asymptotic behavior as $C \to 0$ gives $x_1^* \sim \frac{\alpha_1}{\mu}[w_1 + w_2 + \frac{w_3}{2\sqrt{\alpha_1 \hat{\underline{b}}_1}}]$ and $x_\ell^* \sim \frac{w_1 \alpha_\ell}{\mu}$. Taking the ratio yields $\frac{x_1^*}{x_\ell^*} \sim \frac{\alpha_1}{w_1 \alpha_\ell}[w_1 + w_2 + \frac{w_3}{2\sqrt{\alpha_1 \hat{\underline{b}}_1}}]$. The text incorrectly states the limit as $\frac{\alpha_1}{\alpha_\ell}[w_1 + w_2 + \frac{w_3}{2\sqrt{\alpha_1 \hat{\underline{b}}_1}}]$, missing the w_1 factor in the denominator.

Zero lower bound contradiction: Proposition 2 (p. 2457) relies on the budget C being extremely large. For strategic substitutes, the optimal large-budget intervention targets the last eigenvector (u^n), which assigns opposite signs to most pairs of adjacent nodes. A large-budget intervention $b_i - \hat{b}_i = \sqrt{C} u_i^n$ will involve massive negative shifts for roughly half the network. For any finite status quo $\hat{b}$, a sufficiently large C will drive the standalone returns b_i, and consequently the equilibrium actions a_i^*, into negative territory. While the authors briefly mention non-negativity constraints for small budgets (p. 2455), their asymptotic limits in Proposition 2 are mathematically incompatible with the standard economic assumption of non-negative actions.

Linear-quadratic framework limitation: The entire spectral approach relies on the linear-quadratic framework and the resulting linear best-response functions, characterized by the linear system $[I - \beta G]a^* = b$ (p. 2448). This is a foundational limitation. If real-world strategic interactions exhibit non-linearities, thresholds, or diminishing returns, the system cannot be solved with matrix inversion, and the clean relationship between eigenvalues and amplification factors breaks down.

Limited robustness for large budgets under general costs: The article's striking result that large-budget interventions should be "simple" (proportional to a single eigenvector) depends critically on a specific quadratic cost function. While the authors present a generalization to other cost functions in the supplement (Section OA3. 3), this generalization is only proven to hold for the case of small budgets ($C \to 0$), as stated in Proposition OA1 (p. S14). The lack of a corresponding proof for large budgets suggests the simplicity result may not hold under general cost structures.

Extreme large budget threshold: The article concludes that for large budgets, the optimal policy is a "simple" intervention proportional to a single principal component (Propositions 1 and 2). However, the threshold for what constitutes a "large" budget can be astronomically high, scaling inversely with the squared spectral gap. The authors acknowledge this extreme scaling in footnote 20 (p. 2456), noting a case where the required budget is "About 125 times larger than $\|\hat{b}\|^2$". While the mathematical limit holds, for any realistic policy budget, the optimal intervention will likely remain a complex mixture of components.

Precludes multiple equilibria: Assumption 2 dictates that the spectral radius of βG is less than 1 (p. 2449), ensuring a unique and stable Nash equilibrium. This restricts the framework to environments where interventions have strictly bounded ripple effects, precluding the analysis of scenarios where interventions act as coordination devices to tip a system between multiple equilibria.

Quadratic cost symmetry: The cost function $K(b, \hat{b}) = \sum (b_i - \hat{b}_i)^2$ (p. 2449) is perfectly symmetric, meaning it costs the planner the exact same amount to increase an agent's returns (a subsidy) as it does to decrease them (a tax). In reality, taxes generate revenue and have entirely different political and administrative cost structures than subsidies.

Rotational invariance of variance costs: In Section 5.2, Assumption 5(b) states that the cost of inducing a variance-covariance matrix of shocks is invariant to orthogonal rotations (p. 2462). Economically, this implies that inducing perfectly correlated macro-shocks costs the same as inducing perfectly anti-correlated micro-shocks between neighbors. The administrative and informational costs of the latter would be vastly higher in reality.

Local public goods inequality: In the Local Public Goods example (p. 2450), because the game features strategic substitutes and the objective is transformed to minimize the sum of squared actions, the optimal policy (targeting the last eigenvector) actively maximizes inequality in effort by forcing one set of nodes to work while discouraging their neighbors.

Variance control policy lacks realistic instruments: Proposition 4 (p. 2462) characterizes an optimal "variance control" policy where the planner manipulates the covariance matrix of individuals' private incentives. This is a profoundly abstract concept, and it is exceedingly difficult to map this mathematical result to any realistic, deployable policy instrument.

Simple interventions are informationally complex: The article defines interventions proportional to global eigenvectors as "simple" (Definition 2, p. 2457). While mathematically rank-1, computing these vectors requires perfect, centralized knowledge of the entire network topology. From a policymaker's perspective, this is highly complex and informationally demanding.

Lack of analysis on noise: Spectral methods are highly sensitive to small perturbations in matrix entries. The article assumes the planner has perfect knowledge of the network G and does not analyze how robust the optimal policy is to inevitable measurement errors or misspecification of the network topology.

Lack of comparison to simple heuristics: The article derives a mathematically complex optimal intervention but does not benchmark its performance against simpler, more intuitive heuristics (e.g., targeting high-degree nodes). Without knowing the marginal welfare gain of the optimal spectral policy over a naive heuristic, it is difficult to assess the practical value of the article's complex informational requirements.

Incomplete information certainty equivalence: Proposition 3 (p. 2461) shows that the optimal mean-shifting policy under uncertainty is identical to the deterministic case. While presented as an insight for policy under uncertainty, it is debatable whether this is a general principle or simply a standard certainty equivalence result driven mechanically by the linear-quadratic structure of the model.

Cosine similarity obscures magnitude: Theorem 1 (p. 2454) characterizes the optimal intervention entirely in terms of cosine similarity (angles), which abstracts away from the absolute magnitude of the financial allocation that policymakers care about.

Notation errors: Several purely cosmetic notation errors appear in the text. On p. 2465, the Lagrangian contains typographical errors: the objective term is written as $\underline{\hat{b}}_\ell$ instead of $\underline{\hat{b}}_\ell^2$, and the constraint term is written as $\hat{b}_\ell^2$ instead of $\underline{\hat{b}}_\ell^2$ (missing the underline). Similarly, in Equation (11) on p. 2465, the first-order condition writes the leading term as $2\hat{b}_\ell^2$ instead of $2\underline{\hat{b}}_\ell^2$, missing the underline. In Lemma OA2 part 2 (p. S10), the formulas for the derivative also miss the underlines on the $\hat{b}_1^2$ and $\hat{b}_\ell^2$ terms. Finally, in Equation (9) on p. 2461, the transpose is incorrectly placed inside the parenthesis as $(\underline{a}^\top)^*(\underline{a}^*)$ instead of $(\underline{a}^*)^\top(\underline{a}^*)$.

Presentation and clerical issues: There are a few minor presentation issues in the supplementary material. Proposition OA2 is stated on p. S16, but no proof is provided in the text or supplementary material. In Lemma OA3 (p. S15), G is used to denote the asymptotic limit of the cost function, creating a notation clash with the global definition of G as the network adjacency matrix. On p. S11, the sentence "Suppose, toward a contradiction, that $x_1^* < 0$." is accidentally duplicated. Finally, in the proof of Proposition OA3 (p. S17), there is an undefined and mathematically unnecessary reference to "U" when discussing the boundary of set E.

Standard modeling limitations: The analysis is based on a one-shot, simultaneous-move game assuming the network of interactions G is fixed and exogenous. This static framework ignores crucial dynamic aspects of network interventions. Powerful targeted interventions are likely to induce behavioral responses that alter the network structure itself, such as forming new links with subsidized groups or severing links due to induced local competition. Furthermore, the assumption of a benevolent utilitarian planner maximizing the sum of utilities is a normative choice. Real-world policymakers often optimize for equity, target specific subgroups, or face political constraints.

Future Research

Incorporating non-negativity constraints: Future work could explicitly incorporate non-negativity constraints on actions into the planner's optimization problem. Analyzing how binding constraints alter the optimal spectral targeting would clarify whether the simplicity results hold for large budgets in economically realistic settings.

Robustness to network misspecification: Researchers could analyze the sensitivity of spectral targeting to measurement errors or missing links in the network topology. Benchmarking the performance of the optimal spectral policy against simpler, localized heuristics (such as targeting high-degree nodes) would help determine the marginal value of acquiring perfect global network information.

Asymmetric cost structures: Future models could introduce asymmetric cost functions to reflect the differing political and administrative realities of taxing versus subsidizing. Exploring how asymmetric costs distort the alignment between optimal interventions and the network's principal components would provide a more realistic guide for policymakers.

Beyond linear best responses: Future research could extend the principal component approach to environments with non-linear best responses or multiple equilibria. Investigating whether the spectral insights hold locally around specific equilibria or can be used to design interventions that tip a system between equilibria would significantly broaden the framework's applicability.

Copyediting

The manuscript presents an elegant theoretical framework for targeting interventions in network games, offering a novel mathematical decomposition of network spillovers. However, there are several mathematical and algebraic errors in the supplementary material that need to be addressed to ensure the proofs are robust. Additionally, acknowledging the practical limitations of the linear-quadratic framework and the informational requirements of the proposed interventions would strengthen the paper's positioning.

- **p. S14** "$\max_b C^{-1}\Delta(C^{1/2}\breve{y})$" As $C \to 0$, the term $C^{-1/2}$ in the expansion diverges to infinity, making the limit function undefined and invalidating the use of Berge's theorem of the maximum. Consider changing the scaling factor of the objective function to $C^{-1/2}$ to resolve the divergent limit and ensure the theorem can be properly applied.

- **p. S6** "$w_1 = 1 + m_2 + m_5 + (n-1)m_4, w_2 = nm_5(n-2)$" The derivation of the aggregate welfare coefficients contains algebraic errors. The base coefficient for the sum of squared actions should be 1/2, not 1, and w_2 should utilize the parameter m_4 instead of m_5. Revise the coefficients to "$w_1 = 1/2 + m_2 + m_4 + m_5(n-1)$" and "$w_2 = m_4(n-2)$".

- **p. S8** "4. Suppose the game has strategic complements, $\beta > 0$. In the limit as $C \to \infty$" The premise of the theorem allows for $w_1 < 0$ and $\sum_{\ell=2} \underline{\hat{b}}_\ell^2 > C$. If $w_1 < 0$, the budget C is strictly bounded from above, making a limit to infinity logically contradictory. Explicitly restrict the asymptotic limits in parts 4 and 5 to the case where $w_1 \geq 0$.

- **p. S19** "$K(y) = \frac{1}{2}\sum_i \mathbf{1}_{y_i>0} \int_{a_i(y)-y_i}^{a_i(y)} s_i^1(\tau_i)d\tau_i + \sum_i(1-\mathbf{1}_{y_i>0}) \int_{a_i(y)}^{a_i(y)+|y_i|} s_i^0(\tau_i)d\tau_i$" The integral for $y_i > 0$ evaluates to $\frac{1}{2}y_i^2$. With the leading 1/2 factor, the first summation evaluates to $\frac{1}{4}\sum_{y_i>0} y_i^2$, which contradicts the final equality of $\frac{1}{2}\sum_i y_i^2$. Remove the erroneous leading 1/2 factor from the first summation in the cost of intervention formula.

- **p. S14** "where $\tilde{\kappa}(y) = y^2 + c|y|^3 e^y + c'y^4$, with c and c' being arbitrary constants." If c' is an arbitrary negative constant, the cost function will become negative as $y \to -\infty$, violating the non-negativity requirement in Assumption OA2 part (3). Explicitly constrain the arbitrary constant c' to be non-negative (e.g., "$c' \geq 0$").

- **pp. S9-S10** "$x_1 \in [-C/\hat{\underline{b}}_1, C/\hat{\underline{b}}_1]$" The condition $C(x_1) \geq 0$ implies $x_1^2 \leq C/\hat{\underline{b}}_1^2$, which simplifies to $|x_1| \leq \sqrt{C}/|\hat{\underline{b}}_1|$. The current interval bounds are missing the square root on C. Add the missing square root to C in the bounds of the feasible interval for x_1, revising to "$x_1 \in [-\sqrt{C}/\hat{\underline{b}}_1, \sqrt{C}/\hat{\underline{b}}_1]$".

- **p. S4** "$a_i = \frac{\tilde{b}_i}{1+\gamma} + \frac{\tilde{b}_i+\gamma}{1+\gamma}\sum_j g_{ij}a_j$" Taking the derivative of the utility function with respect to a_i yields $\tilde{\beta} + \gamma$ in the numerator of the second term, not $\tilde{b}_i + \gamma$. Change the numerator of the second term in the first-order condition from $\tilde{b}_i + \gamma$ to $\tilde{\beta} + \gamma$.

- **p. S6** "$u_i^1(G) = \sqrt{n}$ for all i" The principal components are defined as orthonormal, meaning $\|u^1\| = 1$. If $u_i^1(G) = \sqrt{n}$, the norm would be n, violating the normalization. Correct the normalization of the eigenvector from $\sqrt{n}$ to $1/\sqrt{n}$.

- **p. S5** "$\hat{U}_i(a, G) = a_i(b_i + \sum_j g_{ij}a_j) - \frac{1}{2}a_i^2$" To yield the correct equilibrium condition $a_i = b_i + \beta\sum_j g_{ij}a_j$, the spillover term is missing the strategic interaction parameter β. Add the missing parameter β to the spillover term, revising to "$\hat{U}_i(a, G) = a_i(b_i + \beta\sum_j g_{ij}a_j) - \frac{1}{2}a_i^2$".

- **p. S5** "$W(b, G) = \frac{1}{2}(a^*)^\top a^* - n\gamma\sum_i a_i^*$" Summing the individual utility $U_i(a, G) = \hat{U}_i(a, G) - \gamma\sum_{j\neq i} a_j$ over all i yields an externality term of

$-\gamma(n-1)\sum_i a_i$. The coefficient $n\gamma$ is contradictory. Change the coefficient for the externality term in the aggregate welfare equation from $n\gamma$ to $\gamma(n-1)$.

- **p. S17** "This is the square root of the aggregate equilibrium utility in the game with $b = 1_i$, that is, the squared root of $a(1_i)^\top a(1_i)$." The aggregate equilibrium utility is defined as $\frac{1}{2}\|a^*\|^2$. Therefore, the square root of the aggregate equilibrium utility should include a $1/\sqrt{2}$ factor. Add the missing $1/\sqrt{2}$ factor to the definition of the square root of the aggregate equilibrium utility.

- **p. S13** "$\underline{a}_\ell^* = \frac{1}{s_\ell}\underline{b}_\ell^2$" Solving the equilibrium condition $S\underline{a}^* = \underline{b}$ for the ℓ-th component yields $\underline{a}_\ell^* = \frac{1}{s_\ell}\underline{b}_\ell$. The squared term on $\underline{b}_\ell$ is erroneous. Remove the erroneous square from $\underline{b}_\ell$ in the singular value decomposition equilibrium action formula.

- **p. S8** "$\frac{x_1^*}{x_\ell^*} \to \frac{\alpha_1}{\alpha_\ell}\left[w_1 + w_2 + \frac{w_3}{2\sqrt{\alpha_1 \underline{\hat{b}}_1}}\right]$" Taking the ratio of the asymptotic behaviors $x_1^* \sim \frac{\alpha_1}{\mu}[\dots]$ and $x_\ell^* \sim \frac{w_1\alpha_\ell}{\mu}$ yields a w_1 factor in the denominator that is currently missing. Add the missing w_1 factor to the denominator in the asymptotic limit ratio, revising to "$\frac{\alpha_1}{w_1\alpha_\ell}$".

- **p. 2465** "$\mathcal{L} = w\sum_{\ell=1}^n \alpha_\ell(1+x_\ell)^2\hat{b}_\ell + \mu\left[C - \sum_{\ell=1}^n \hat{b}_\ell^2 x_\ell^2\right]$" The variables representing the principal components of the status quo should be underlined as $\underline{\hat{b}}_\ell$ to maintain consistent notation. This error also appears in Equation 11 on the same page and in Lemma OA2 on p. S10. Add the missing underlines to all instances of $\hat{b}_\ell$ in the Lagrangian, Equation 11, and Lemma OA2.

- **p. 2461** "$w\mathbb{E}[(\underline{a}^\top)^*(\underline{a}^*)]$" The transpose is incorrectly placed inside the parenthesis, which is mathematically imprecise for the intended vector operation. Move the transpose outside the parenthesis in Equation 9 to correctly read "$(\underline{a}^*)^\top$".

- **p. S17** "On the other hand, b^* must be on the (elliptical) boundary of E because U is strictly increasing in each component" There is an undefined and mathematically unnecessary reference to U when discussing the boundary of set E, which causes confusion. Remove the undefined reference to U when discussing the boundary of set E.

- **p. S15** "converges uniformly to $k\|z\|^2$, as $C \downarrow 0$, where $k > 0$ is some constant. We call the limit G." Using G to denote the asymptotic limit of the cost function creates a notation clash with the global definition of G as the network adjacency matrix. Change the variable name G to something else (e.g., $\tilde{G}$ or H) in Lemma OA3 to resolve the notation clash.

- Consider adding a dedicated discussion section that explicitly acknowledges the practical limitations of the model, such as the informational burden of computing global eigenvectors and the assumption of symmetric costs, framing them as natural next steps for future research.

Proofreading

The following errors were identified:

- **Page 5**: “**sufficienctly**” -> “**sufficiently**” (Spelling error)
- **Page 11**: “**Suppose, toward a contradiction, that $x* 1 < 0$. Suppose, toward a contradiction, that $x* 1 < 0$.**” -> “**Suppose, toward a contradiction, that $x* 1 < 0$.**” (Repeated sentence)
- **Page 16**: “**for al i**” -> “**for all i**” (Spelling error)

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